Reliability-

The reliability of the product or service is its ability to retain its quality over a period of time.

Quality and reliability are inter-related.

Definition:
Reliability is the **probability** that the product, process or service perform its **intended function** for a **specified interval** under stated **operating condition**.
Probability- measurable (between 0 to 1)
Intended function- e.g purpose of fridge
Specified interval- Reliability decreases with time; interval can have any units like distance, cycles etc
Operating condition- environment (climate, packaging, storage, transportation, the user, maintenance)

For things that cannot be repaired the definition shrinks
Reliability is the **probability** that the product perform its **intended function** under stated **operating condition.**

Why Improving Reliability

- Higher failure cost limit profit
- Reduced warranty problems (less premature failures)- legal implications
- Less downtime for process industries
- Cutbacks in output result in economic loss
- Reflection on producers image (A rapport)

Buyers- use reliability when comparing alternatives
Sellers- use reliability in determining the Price (Cost + Profit)

Potential ways to improve Reliability

- Improve component design
- Improve production and/or Assembly techniques
- Improve testing
- Improve preventive maintenance procedures
- Improve User education
- Use redundancy – use of backup component
- Improve system design- simplify the system (less no of component and interfaces)

Suppose a large number of identical items (same manufacturer, design, batch number, environment etc) are put on test together, they will **not** fail at the same time. Lifetimes are distributed.

A histogram is shown as Fig 1.1 'x-axis' mission duration or ageing parameter, 'y-axis' proportion. The histogram describes the amount of failures as a function of time.- **f(t) is a failure distribution**
Approximately two percent of the item fails in time between 0 and 1 or first hour.
In second hour 3 %, and in third hour 5 % etc

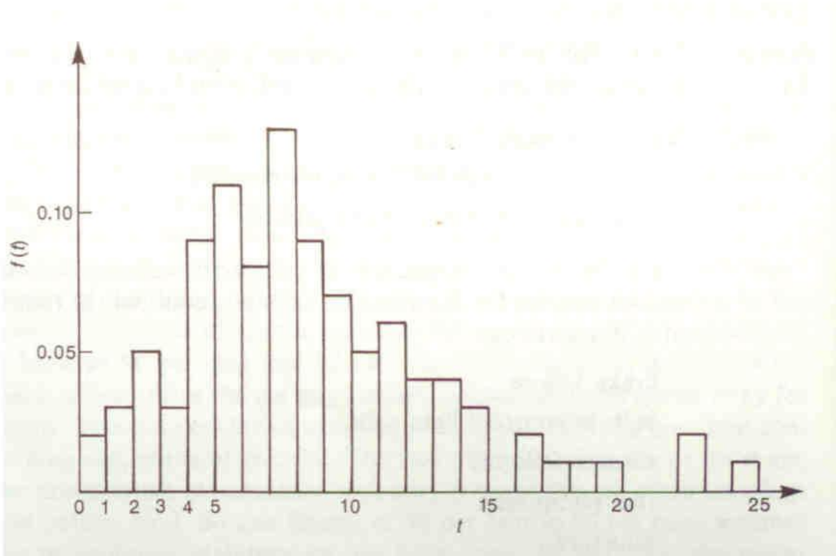


Fig. 1.1 Histogram of lifetimes.

In many cases the proportion of failure in specified interval is of less interest, but rather the total proportion of the components have failed up to a time or until a certain interval has reached. The answer is from cumulative histogram. (In Fig 1.2 height of each bar is the sum of heights of bars in Fig 1.1)

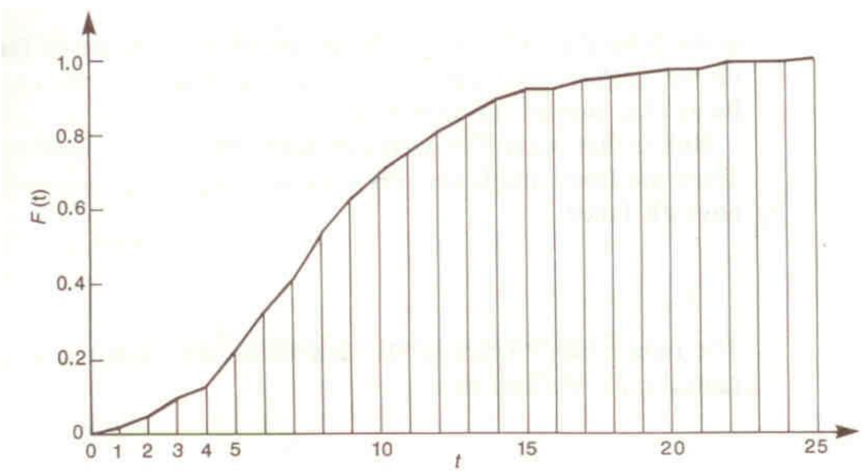


Fig. 1.2 Cumulative graph of lifetimes: the Failure Function.

After 3 hours 10 % have failed and 90 % survived. So the reliability decreases with time. So there are fewer and fewer survivors as time pass, until eventually they have all failed as shown in fig 1.3.

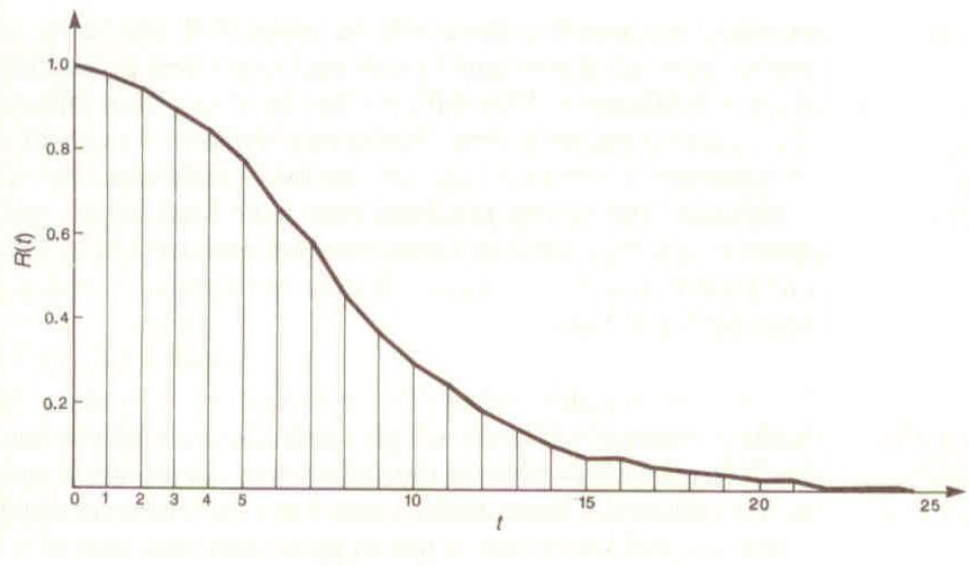
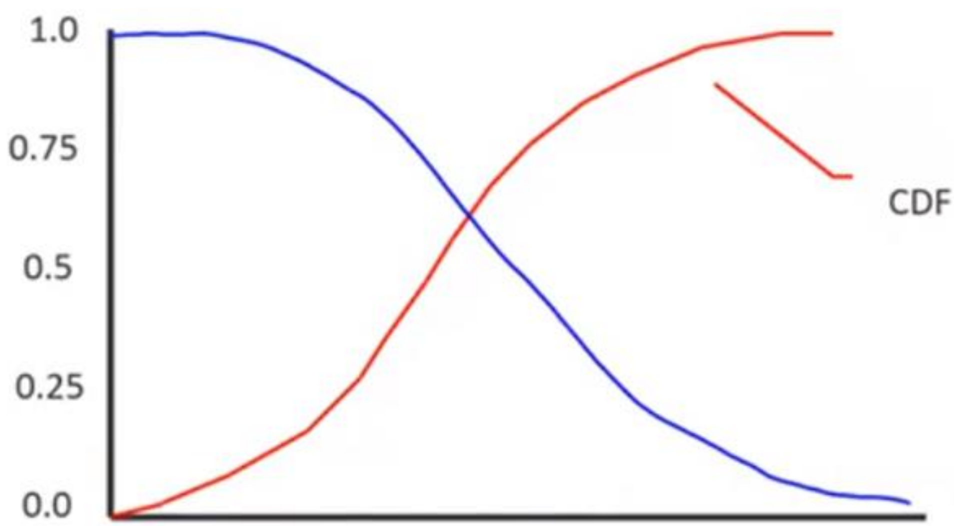


Fig. 1.3 The reliability function.

Reliability Function

- Reliability function is the complement of CDF



The above histograms are for the **discrete** data. When large number of trials to be made the width of each bar in the histogram will shrink and the distribution becomes **continuous** (Actually time is a continuous entity) as shown in Fig 2.2. The curve shows the distribution of the lifetimes and the function *f(t)* is called the probability density function or **pdf**. In reliability works it is also known as the **failure density function.** The pdf, denoted by *f(t),* indicates the failure distribution over the entire time range. The larger the value of *f(t),* the more failures that occur in a small interval of time around t.

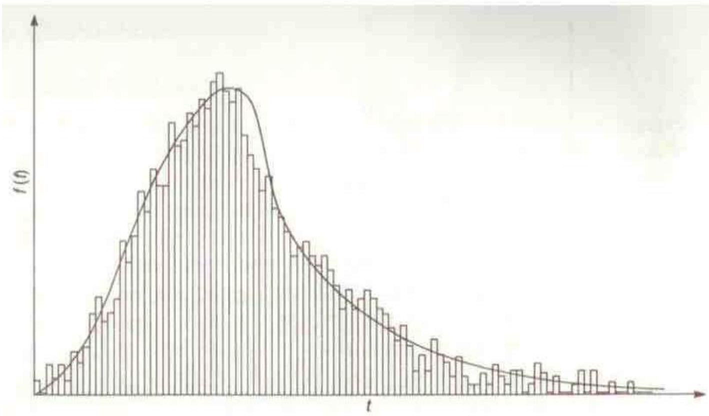


Fig. 2.2 Histogram of lifetimes and probability density function.

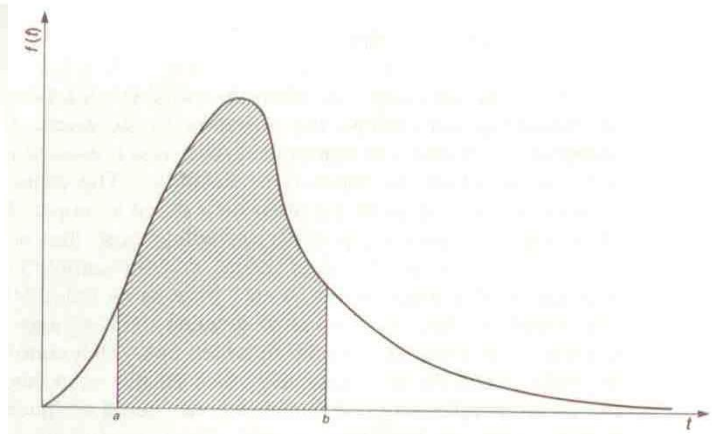


Fig. 2.3 Probability density function.

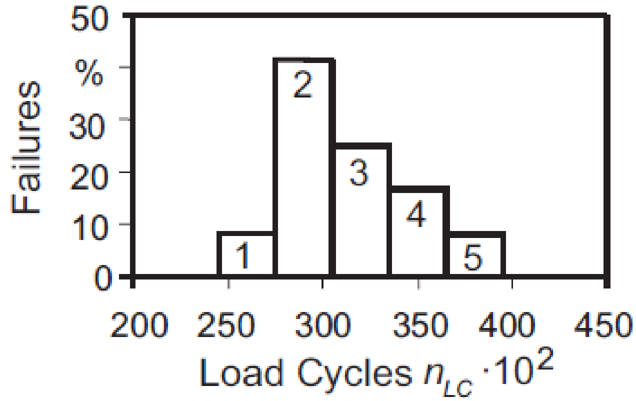
The area under the curve represented by *f(t)* gives the probability. Mathematically the probability of a component failing between times *a* and b is

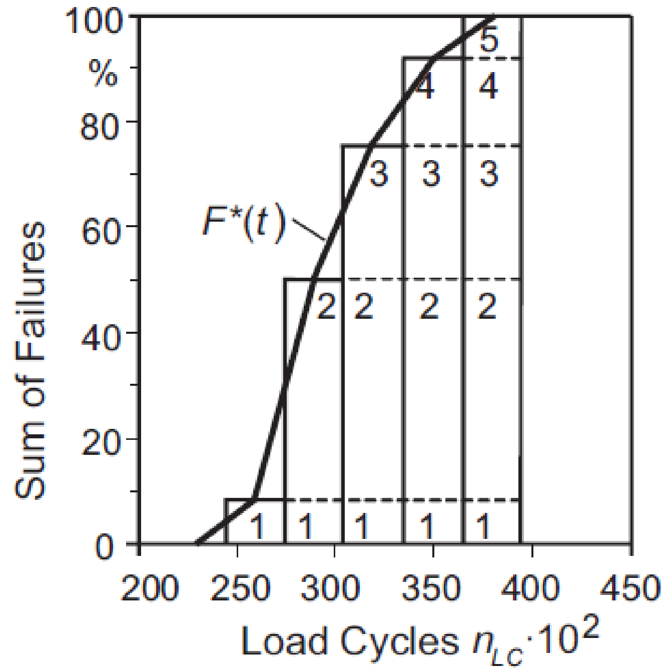
$$P(a \leq t \geq b) = \int_a^b f(t)dt$$

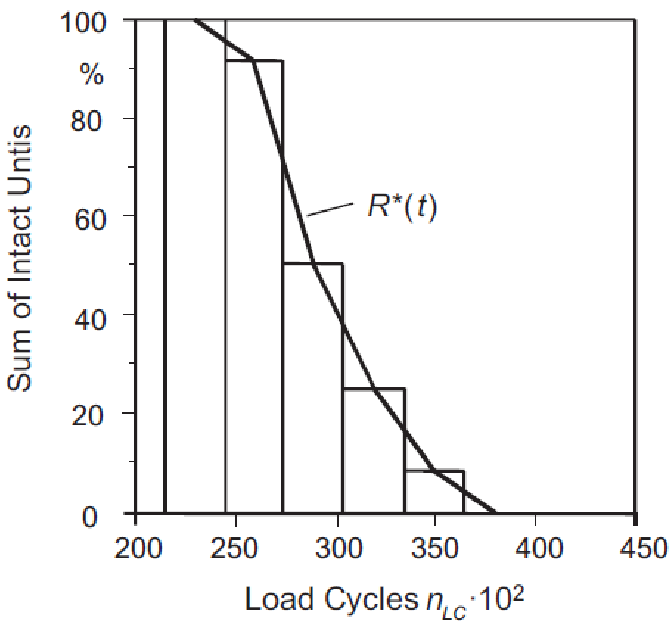
Fig 2.3 shows that the probability is the area under the curve. It is clear from this graph that the area under the curve must be unity between 0 and infinity.
Mathematically we can write as

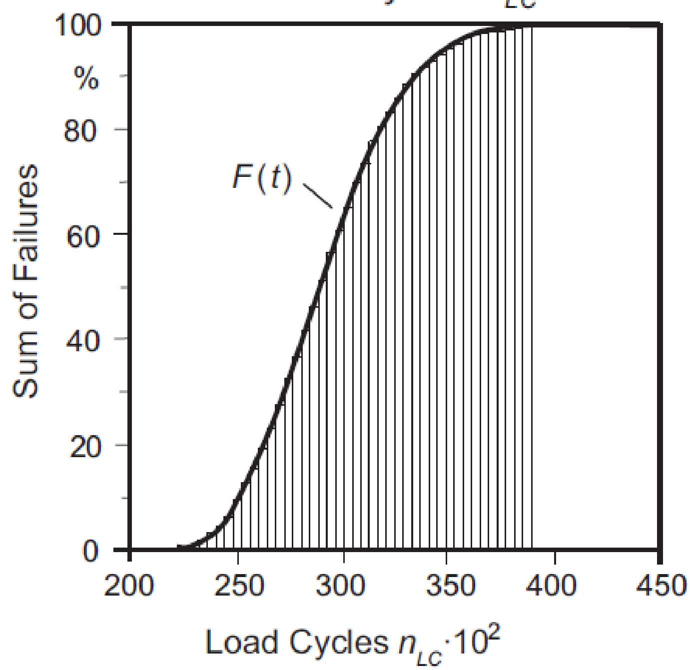
$$\int_0^\infty f(t)dt = 1$$

Another example









Relationship between pdf *f(t), Cumulative distribution function (cdf) F(t)* and *R(t)*

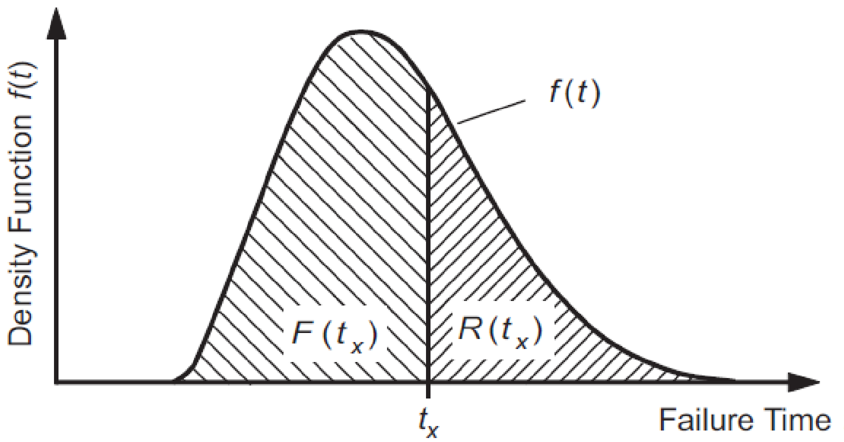
$$F(t) = \int f(t)dt$$

Thus the density function is the derivative of the distribution function

$$f(t) = \frac{dF(t)}{dt}$$

In reliability theory the distribution function *F(t)* is known as **failure probability which** describes the sum of failures as a function of time. Thus $F(t_x)$ is the probability that the component has failed by time t_x or time to failure is less that t_x or in short the population fraction failing by time t.

$$F(t_x) = \int_0^{t_x} f(t)dt$$



Similarly $R(t_x)$ is the probability that the item has **not** failed to lifetime or time to failure is greater than t_x.

$$R(t_x) = \int_{t_x}^{\infty} f(t)dt$$

The survival probability *R(t)* is complement to the failure probability *F(t).* Thus

$$R(t) = 1 - F(t)$$

In reliability theory the survival probability is called Reliability *R(t).*
R(t) begins with 100 % since no failures have occurred at t=0 and then decreases monotonically and ends with 0 % because all units have failed.

Another example of human deaths

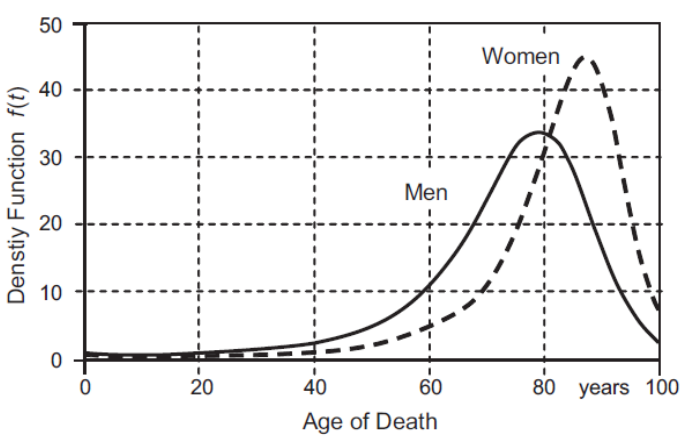


Figure 2.9. Density function $f(t)$ of human deaths

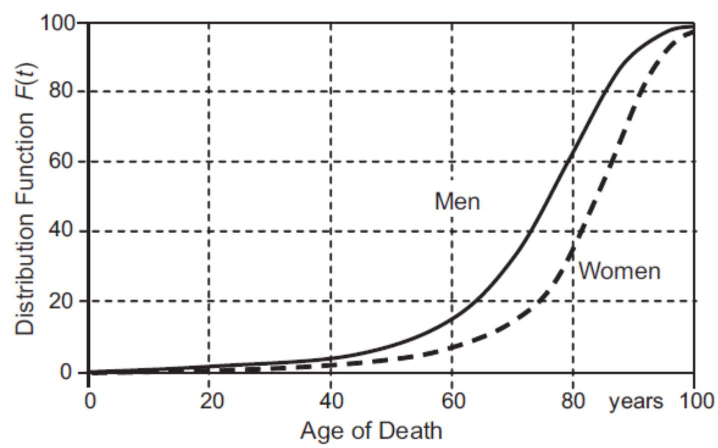


Figure 2.13. Failure probability $F(t)$ for human deaths

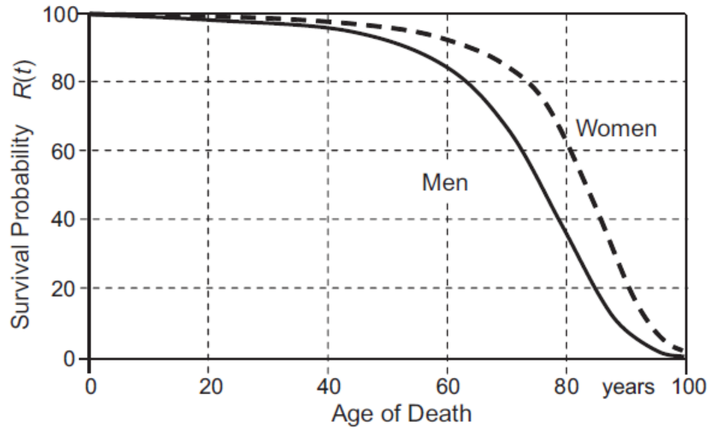


Figure 2.17. Survival probability $R(t)$ for human beings

Another example of 6 gear commercial vehicle transmission

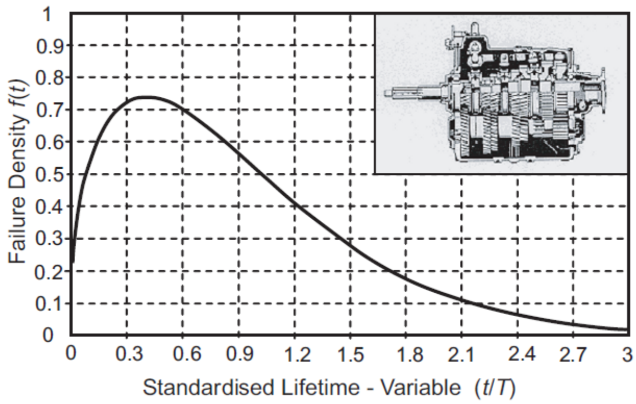
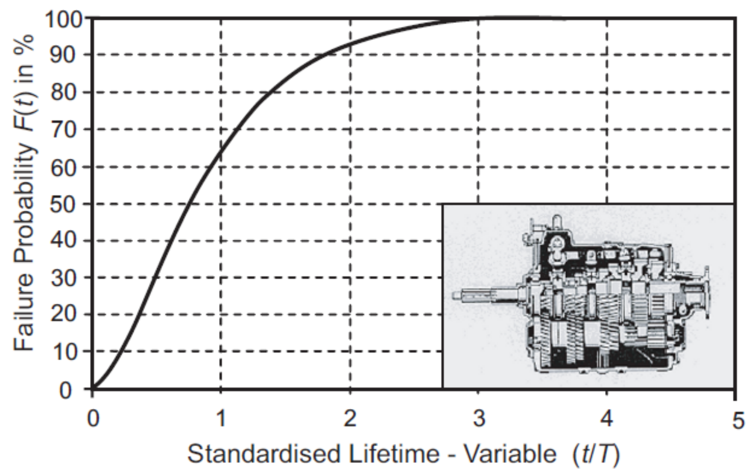
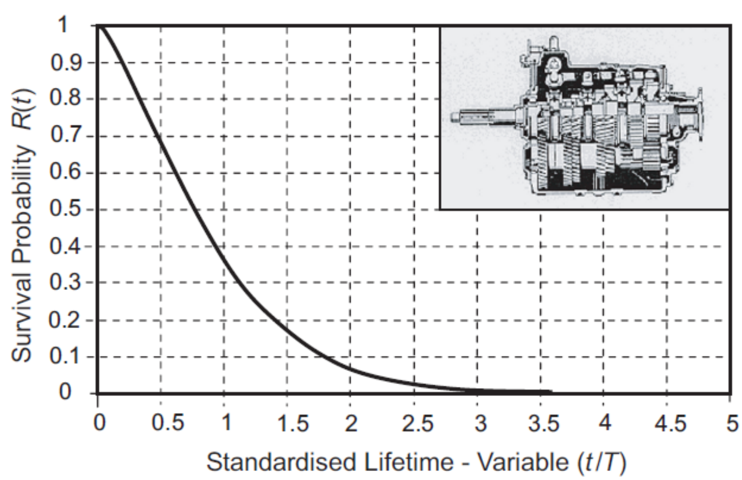


Figure 2.8. Failure density $f(t)$ of a 6 gear commercial vehicle transmission





Failure rate or Hazard rate l(t)

The notion of **aging** describes how a unit improves or deteriorates with its age. Aging is usually measured based on the term of a failure rate function. *l(t)* is a time dependent failure rate which is defined as

$$\lambda(t) = \frac{f(t)}{R(t)}$$

Let '*t*' be the non-negative continuous random variable, denoting the time to failure (useful life) of a component.

$$\left\{\begin{array}{l} probability\ that\ failure \\ takes\ place\ at\ a\ time \\ between\ T\ and\ T+\Delta T \end{array}\right\} = P(T < t \leq T + \Delta T) = f(t)\Delta T$$

Where *f(t)* is the pdf or failure probability per unit time.

$$\left\{\begin{array}{l} probability\ that\ failure \\ takes\ place\ at\ a\ time \\ less\ than\ or\ equal\ to\ T \end{array}\right\} = P(t \leq T) = F(t)$$

$$\left\{\begin{array}{l} probability\ that\ a\ component \\ operates\ without\ failure\ for \\ a\ length\ of\ time\ T \end{array}\right\} = P(t > T) = R(t)$$

$$\left\{\begin{array}{l} probability\ that\ a\ component \\ will\ fail\ at\ some\ time \\ T < t \leq T + \Delta T,\ given\ that\ it\ has\ not \\ yet\ failed\ at\ t = T \end{array}\right\} = P(T < t \leq T + \Delta T \mid t > T) = \lambda(t)\Delta T$$

$$P(T < t \leq T + \Delta T \mid t > T) = \frac{P\{(T < t \leq T + \Delta T) \cap (t > T)\}}{P(t > T)}$$

$$= \frac{P(T < t \le T + \Delta T)}{P(t > T)}$$

$$\lambda(t)\Delta T = \frac{f(t)\Delta T}{R(t)}$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$

So ***l(t)*** is "the failure rate per unit time at time '*T*', conditional on survival until time T"
The graphical representation of the failure rate is shown in Figure 2-19

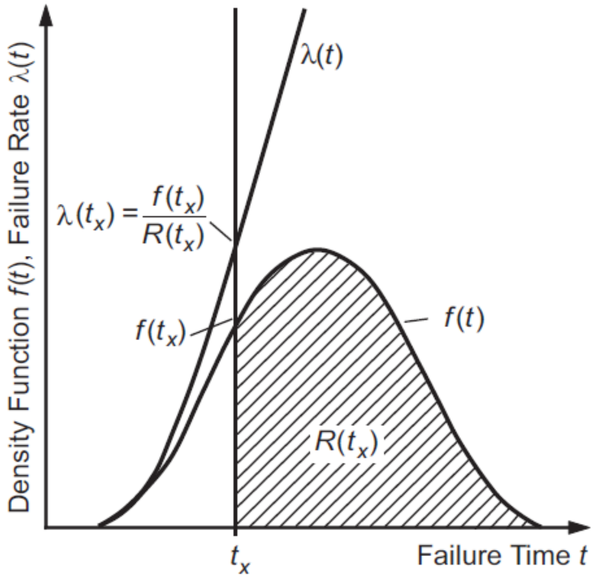


Figure 2.19. Determination of the failure rate out of the density function and survival probability

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\frac{dF(t)}{dt}}{R(t)} = \frac{\frac{d[1-R(t)]}{dt}}{R(t)} = \frac{\frac{-dR(t)}{dt}}{R(t)}$$

$$\lambda(t)dt = -\frac{dR(t)}{R(t)}$$

Integrate

$$\int_0^t \lambda(t)dt = -\int_1^{R(t)} \frac{dR(t)}{R(t)} = -\ln R(t)$$

$$R(t) = e^{-\int_0^t \lambda(t)dt}$$

This can be used to obtain a component reliability for any known failure time distribution.
If any of the four quantities *f(t), F(t), R(t)* and *l(t)* is given, the other three may be obtained from it.

The integral of the failure or hazard rate is known as cumulative hazard function denoted by H(t)

$$H(t) = \int_0^t \lambda(t)dt$$

Problem1: Assume *l(t)=l* (a constant) and find *f(t), F(t), R(t).*

MTTF(Mean time to failure)

It is the expected time during which the component will perform its function successfully. Mathematically defined as:

$$MTTF = E(t) = \int_0^\infty tf(t)dt$$ where *f(t)* is the failure density function.

$$= \int_0^\infty t\left(-\frac{dR(t)}{dt}\right)dt$$

$$= -\left[t.R(t)\Big|_0^\infty - \int_0^\infty 1.R(t)dt\right]$$

$$MTTF = \int_0^\infty R(t)dt$$

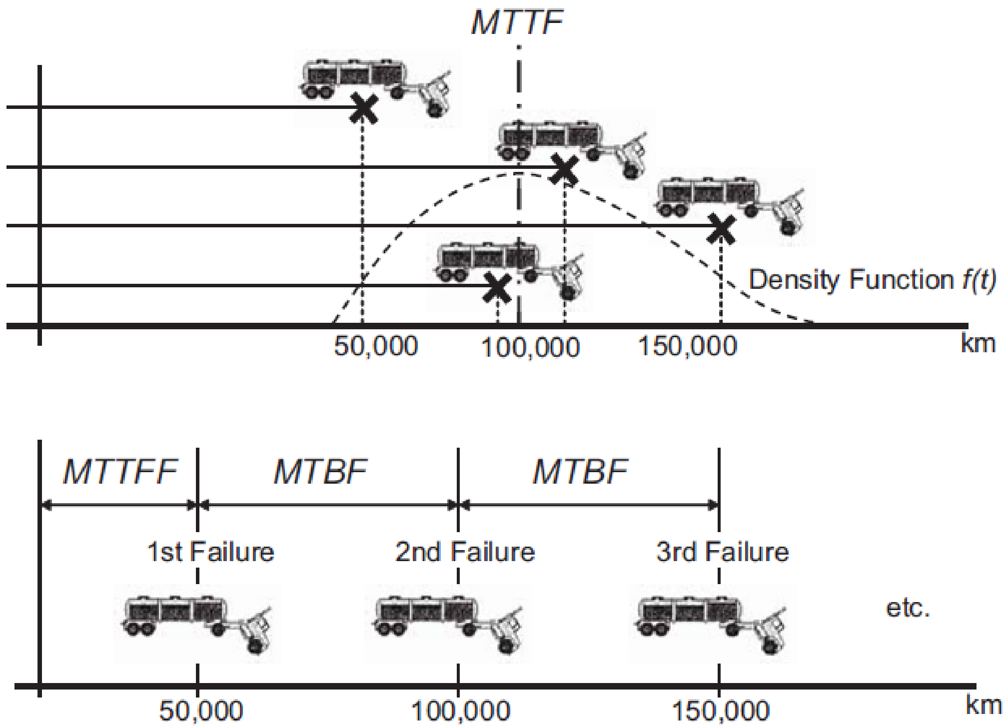


Figure 2.26. Explanation of *MTTF, MTTFF* and *MTBF* on behalf of an example

Life time of repairable component is specified by MTTFF and MTBF
MTTFF=Mean time to first failure, MTBF=Mean time between failure

Under the assumption that the component is as good as new after maintenance, then the next mean time to failure (*MTBF*) is the same as the previous mean time to first failure *MTTFF* after the end of maintenance.